

# Numerical analysis of heat removal enhancement with extended surfaces

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## Abstract

The problem of heat removal is a major issue in modern industry. The reasons for researching in this field are both to increase the performances of the target systems and to reduce the damages that high temperatures can cause. The present trend in high-tech production processes is to seek better performances by means of smaller devices. Aiming at this, it becomes necessary that all the components of the system examined are designed to supply the best possible performance. This paper faces the problem of optimising fins to enhance heat removal. The analysis so conjugates geometrical and thermo-fluid mechanical aspects. The starting point of this activity was a research, based on the Bejan's Constructal Theory, which focused on heat removal enhancement from high temperature surfaces through T-shaped fins. Initially, the same system was here numerically investigated using a CFD code. The performances computed were very similar to the reference ones. This validation allowed to apply the method to new configurations, so to develop systems further on optimised, able to remove higher thermal fluxes in the same processes. Y-shaped profiles were consequently examined, obtained by varying the angle between the two arms of the original T. The idea of performance optimisation as proposed in the reference work, was initially based on the maximisation of the dimensionless thermal conductance. This was here widened to a new definition taking into account thermal efficiency as another parameter of evaluation. It was, in fact, observed that the width reduction, typical of Y-shaped profiles with respect to T-shaped ones, enhance efficiency significantly.

This new approach to heat removal optimisation suggested the realisation of arrays with multiple Y-shaped fins. Each array had the same width of the corresponding optimised T-shaped fin. This choice allowed immediate comparisons, so to evaluate the actual performance enhancements typical of multiple-fin configurations, with respect to previous configurations.

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## 1. Introduction

Every process involving energy is in modern industry strongly affected by the technological constraints due to the dissipations: this unwanted heat generation can significantly reduce the performances. For such a reason the devices and techniques able to remove those thermal energies play a main role. In many applications, however, this process has to happen in presence of very restrictive dimensional constraints. Examples can be found in many applica-

tions within the automotive field [1–3], where innovative engines require enhanced heat removal, and within heating and air conditioning problems, where better performances are often due to more effective heat exchangers. Anyway, it is the field of electronics the one more interested by requirements of dimensional reduction. The recent development in this field, in fact, has permitted the realisation of computing systems able to operate at very high frequencies, thanks to a trend of dimensional reduction of each operating component. The modern microprocessors for computers are realised in fact integrating on a surface of an order of magnitude equal to a square centimetre, a very high number of modular computing units. An increment of performances causes a more elevated dissipation as a consequence of the number of single modules integrated in the CPU. The heat generated, in addition to the risk of physically damaging

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## Nomenclature

$a$	dimensionless parameter, Eq. (5) [-]
$A$	area [m <sup>2</sup> ]
$d$	space between two fin in the array [m]
$h$	heat transfer coefficient [W m <sup>-2</sup> K <sup>-1</sup> ]
$i$	number of modules [-]
$k_1, k_2$	dimensionless ratios [-]
$L$	length [m]
$q$	heat flux [W]
$t$	thickness [m]
$T$	temperature [K]
$V$	volume [m <sup>3</sup> ]
$W$	width [m]
$X(\alpha)$	length of the Y-shaped fin (function of $\alpha$ ) [m]
$X'$	length of the optimal T-shaped fin [m]

### Greek symbols

$\alpha$	angle [°]
$\varphi$	volume ratio of the fin, Eq. (2) [-]
$\lambda$	fin thermal conductivity [W m <sup>-1</sup> K <sup>-1</sup> ]

### Subscript

f	related to the fin
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### Superscript

(*)	dimensionless variables, Eq. (1)
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the electronic components, reduces the global performance of the system so limiting the chance of technical improvements. The importance of ever more effective heat exchangers is so proportional to the advancements of electronics. From a technological point of view, the thermal exchange in electronic components is usually realised with forced convection systems that, in general, consist of a fan coupled to a heat exchanging module made of highly-conductive materials and designed to have an optimal shape in relation to the problem treated [4–6].

Designing a heat exchanger is traditionally made in laboratory, where the shapes chosen are directly verified in function of the parameters of interest. Many works are present in literature. Among them two deserve particular mention: the one in [7], where finned heat exchangers cooled by air in forced convection have been analysed; the one in [8], that focuses on the performance in the air-side of the heat exchangers.

The experimental method, however, tends to be very burdensome both in economic and in temporal terms, as each parameter modification implies a physic re-elaboration of the component being tested. Moreover, because of the unavoidable experimental error, the assessment of each single parameter can be strongly affected by the method employed and so does the final result. Other studies are based on theoretical-numerical methods. They show a more complete insight on the trend of some phenomena, because based on mathematical models created in relation to the punctual characteristics of the system in exam. The works in [9,10] belong to this approach: they both analyse the performance of finned systems depending on the geometries assigned.

Still theoretical-numerical but based on Bejan's Constructal Theory [11], is the main reference work for this research [12]. It presents an analysis of the thermal behaviour of a T-shaped fin, seeking an optimal relationship to show the mutual dependence of the dimensionless geometrical parameters of interest with the one accounting for

performance. The consequent geometrical optimisation of the T-shaped fins implies the maximisation of the thermal performance parameter of reference (the dimensionless conductance), in accordance with the Constructal approach. The latter, as in [12], provides a noticeable numerical precision with respect to the actual data available. A large amount of resources is, anyway, required in the definition of the models presented. This could lead to a certain degree of computational complications, that could sometimes make the practical use of the results not too easy.

In this paper it is presented a numerical approach that gives an alternative or at least a useful integration to the classical experimental method. A CFD code based on the finite element method was employed. Such a software allows to realise a CAD model of the profile to be analysed and to specify its material properties and boundary conditions, so obtaining a complete simulation of the general thermo-fluid mechanical problem. It can be so deduced that this research can face a wide range of parametrical variations, as the physical realisation of the heat exchangers is replaced by virtual models that can be easily modified. One of the most significant advantages of a CFD approach is to make an industrial use of models with high complexity more elementary. The research initially focused on analysing the performances of T-shaped fins so to compare our results with those in [12]. The validation of the CFD approach allowed its application to a different geometry: Y-shaped fins, obtained by varying a new degree of freedom (the angle between the two horizontal arms of the original T) with reference to the optimised system in [12]. The heat removal performance evaluation parameter (the dimensionless conductance) [12], was successively integrated by adding efficiency as a second indicator.

The last optimisation involved a multiple-fin analysis and took into account even a suitably defined "space factor" to reach the optimal final module.

## 2. T-shaped fins

### 2.1. Model definition

The research begins by examining the same computational domain shown in [12]. It has been considered a T-shaped profile (Fig. 1), obtained by joining together three flat slabs, one vertical with a thickness  $t_1$  and length  $L_1$ , and two horizontal with a thickness  $t_0$  and length  $L_0$ . The third dimension  $W$  is chosen to be long enough, compared to  $L_1$  and  $L_0$ , to avoid checkerboard effects and computational overloads. The convection coefficient  $h$  is set constant in the whole domain and the temperatures  $T_1$  of the root and  $T_\infty$  of the external fluid are known. The purpose of this section is to determine the optimal geometrical ratios  $L_1/L_0$  and  $t_1/t_0$  guaranteeing the highest dimensionless thermal conductance  $q_1^*$ , defined as:

$$q_1^* = \frac{q_1}{\lambda \cdot W \cdot (T_1 - T_\infty)}, \quad (1)$$

where  $q_1$  is thermal power flowing through the root of the fin,  $\lambda$  is thermal conductivity, while  $T_1$  and  $T_\infty$  are the temperatures at the root and of the surrounding fluid, respectively.

The geometrical model in input to the numerical simulation software has been realised in a CAD environment and imported later using the IGES file format.

As in the reference work [12], the dimensions of the system are defined based on the ratios  $k_1 = t_1/t_0$  and  $k_2 = L_1/L_0$ . The general design of the fin is so done superimposing some geometrical constraints, such as the “apparent” and the actual volume occupied by the fin.

Consequently, it can be defined the volumetric ratio  $\varphi$  obtained as ratio between the actual volume and the “apparent” volume of the fin:

$$\varphi = \frac{A_f}{A}. \quad (2)$$

It is easy to observe that in the expression just presented it has been used the surface. The reason of such a choice is dependent on the third dimension  $W$ , that is constant for both volumes considered.  $A_f$  and  $A$  are defined respectively as follows:

$$\begin{aligned} A_f &= 2 \cdot L_0 \cdot t_0 + L_1 \cdot t_1, \\ A &= 2 \cdot L_0 \cdot L_1. \end{aligned} \quad (3)$$

From these definitions it is possible to compute  $L_0$ , the only unknown variable

$$L_0 = \frac{1}{2 \cdot \varphi} \cdot t_0 \left( \frac{2}{k_2} + k_1 \right). \quad (4)$$

The other dimensions follow directly, from test to test by the values of  $k_1$  and  $k_2$ . As in [12] the value of  $\varphi$  is set equal to 0.1 and held constant in each series of tests; also  $t_0$  is a constant equal to 1. The boundary conditions are defined starting from the convection coefficient  $h$ , uniform and constant in each domain but variable from case to case in function of the geometry

$$h = \frac{a^2 \cdot \lambda}{2 \cdot \sqrt{A}}. \quad (5)$$

It can be observed that  $h$  is directly proportional to the dimensionless parameter  $a$ , for which Eq. (5) is the definition, in accordance with [12]. This parameter has the physical meaning of putting together heat transfer and geometry. The thermal conductivity  $\lambda$  is chosen equal to  $200 \text{ W m}^{-1} \text{ K}^{-1}$ , value that corresponds to Aluminium.  $T_1$  and  $T_\infty$  are set equal to 373.15 and 293.15 K, respectively. Such a choice does not affect the general applicability of the study as the target parameter  $q_1^*$ , is expressed in dimensionless terms.

### 2.2. Partial results

The first part of this work was aimed at validating the CFD-based method here proposed using Almgöbel and Bejan's results [12].

The temperature distribution was quantitatively different from case to case. It can be observed, anyway, a common qualitative trend similar to that in Fig. 2: the temperature field shows its highest values at the root, decreasing in direction of the lateral sides.

The trends of  $q_1^*$ , performance evaluation parameter, are represented in Fig. 3 in relation to the characteristic ratios  $k_1$  and  $k_2$ .

The highest values of  $q_1^*$  are obtained for values of  $k_2$  in the vicinity of 0.07, independently from the parameter  $k_1$ . This result confirms the very good agreement with the approach shown in [12]. The optimal value for  $q_1^*$  can be found for  $k_1$  equal to 5, whereas in [12] it was for  $k_1 = 4$ : even here, anyway, the values of  $q_1^*$  for  $k_1 = 4$  are very close to the best results.

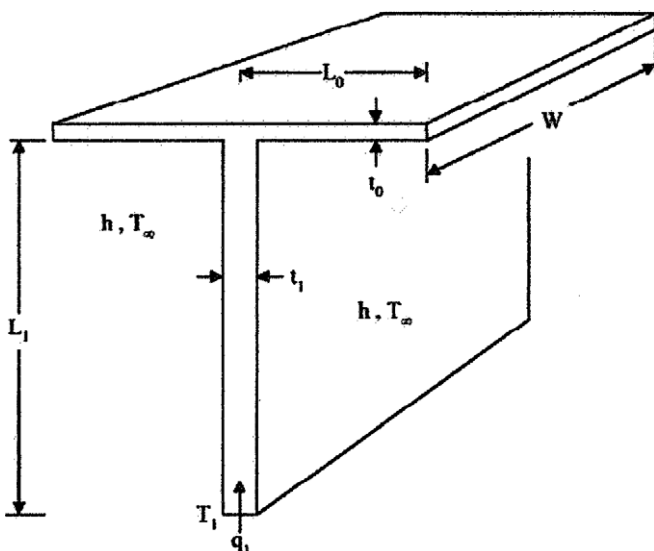


Fig. 1. Geometric definition of the model.

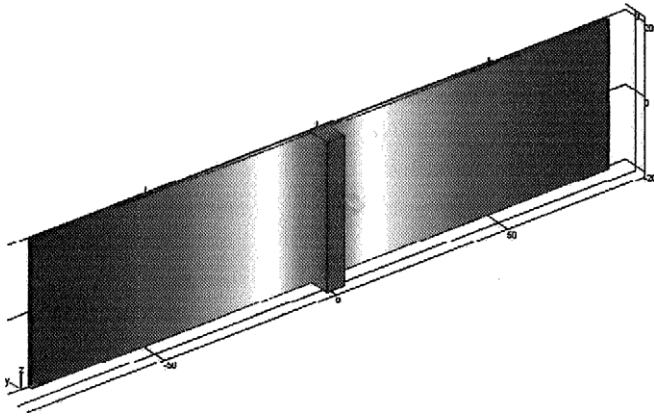


Fig. 2. Graphical output of thermal field.

Starting then from the optimal value for  $k_2$  just found (equal to 0.07) it was analysed the variation of the dimensionless conductance  $q_1^*$  in function of  $k_1$  (Fig. 4). The convex shape of the obtained curve approximates well the corresponding trend presented in [12] and guarantees the presence of a maximum value for  $k_1$  equal to 5, with very close values for  $k_1 = 4$  and  $k_1 = 6$ .

Other parameters that concur to the optimisation of the system, in function of the geometry and of the boundary conditions, are  $a$  and  $\varphi$ , previously defined. The trends presented in Fig. 5 was obtained setting the variables  $k_1$  and  $k_2$  to the optimal values just found, evaluating the performance parameter  $q_1^*$  in dependence of  $a$  and  $\varphi$ .

It can be observed that the lines are about parallel, as in the reference work [12]. The method here proposed has so been validated and can now be applied to more complicated problems to compute further optimisations.

### 3. Y-shaped fins

#### 3.1. Model definition

A first series of tests was done modifying the structure of the T-shaped fins optimised in the previous section. It was evaluated the performance parameter  $q_1^*$  in function of the angle  $\alpha$  between the two “horizontal arms” of the T. In the CAD realisation of the models to be tested, three different cases could be defined.

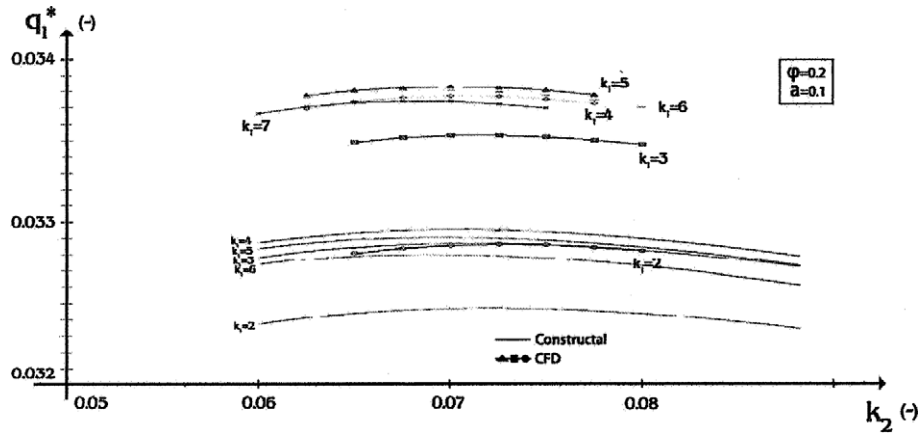


Fig. 3. Dimensionless conductance in function of  $k_1$  and  $k_2$ .

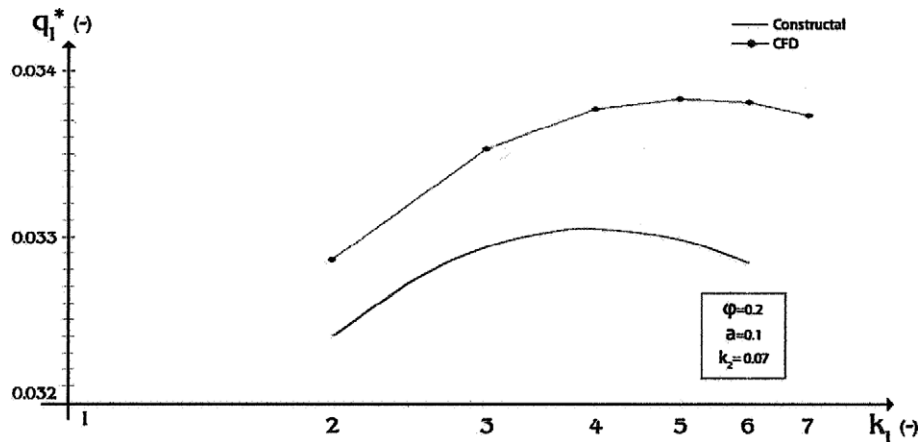


Fig. 4. Variation of  $q_1^*$  in function of  $k_1$ .

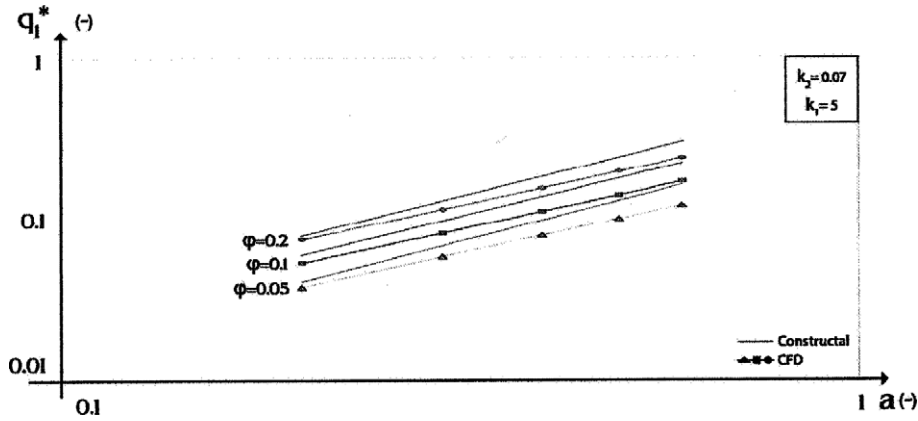


Fig. 5. Variation of  $q_l^*$  in function of  $a$ .

• **CASE I:**  $180^\circ < \alpha < \alpha_{lim}$ .

In this range of values for  $\alpha$  the fin assumes the shape of an arrow (Fig. 6). The case is limited by the border value  $\alpha_{lim}$ , in correspondence to which the arms of the fin become tangent to the plane containing the root of the system.

• **CASE II:**  $\alpha_{lim2} < \alpha < 180^\circ$ .

In this range of value  $\alpha$  confers a Y-shape to the fin. Its smallest value is  $\alpha_{lim2}$ , at which the intersection between the bottom surfaces of the arms is tangent to the plane containing the root of the system (Fig. 7).

• **CASE III:**  $\alpha_{lim3} < \alpha < \alpha_{lim2}$ .

In this range of values, a further reduction of  $\alpha$  with respect to Case II causes an intersection of the two bottom surfaces of the arms to lie under the plane containing the root (Fig. 8).

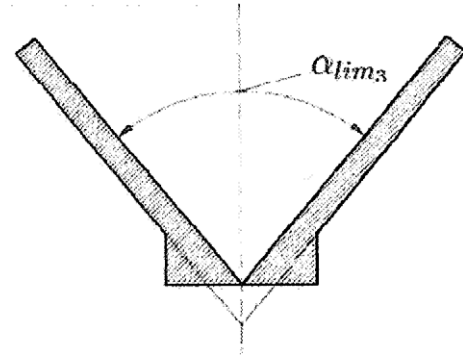


Fig. 8. Case III:  $\alpha_{lim3} < \alpha < \alpha_{lim2}$ .

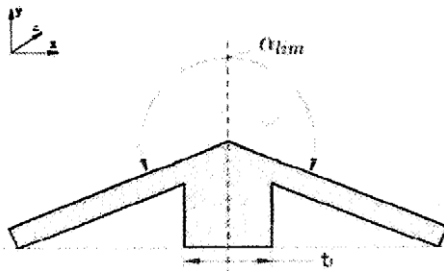


Fig. 6. Case I:  $180^\circ < \alpha < \alpha_{lim}$ .

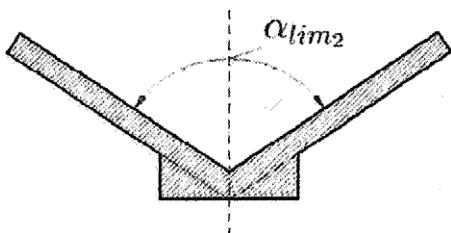


Fig. 7. Case II:  $\alpha_{lim2} < \alpha < 180^\circ$ .

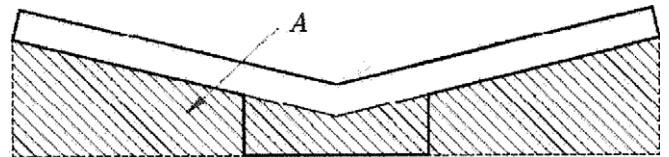


Fig. 9. Subtended surface A.

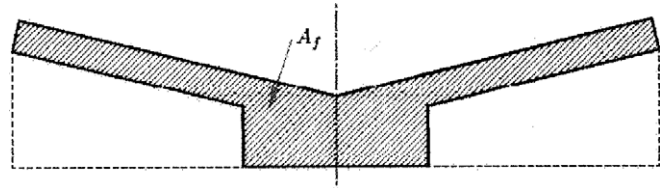


Fig. 10. Cross-sectional surface  $A_f$ .

The volumetric ratio  $\phi$ , defined in Eq. (2) and in Figs. 9 and 10, is now assumed as reference parameter to make the results fully comparables. The values of  $A$  and  $A_f$  can no more be computed by means of Eq. (3); it is now necessary to determine the new relationships considering

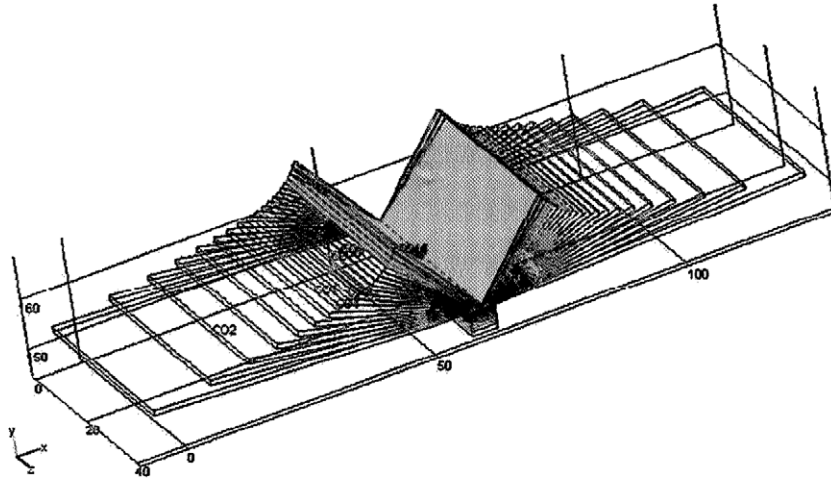


Fig. 11. Geometry depending on  $\alpha$ .

carefully the geometries in Figs. 6–8. The value of  $\varphi$  was assumed constant and equal to 0.1 for each test, as in [12]. This condition made the length  $L_0$  a function of  $\alpha$ , as Fig. 11 shows. The variable  $L_0$  is minimal (see the fully-coloured profile in Fig. 11), in correspondence to

$\alpha = 95^\circ$ . For smaller values of  $\alpha$  it can be observed that  $L_0$  inverts its trend.

The boundary conditions chosen are the same used in the validation Section 2.1. Such a choice was made to analyse the performances just in dependence of  $\alpha$ .

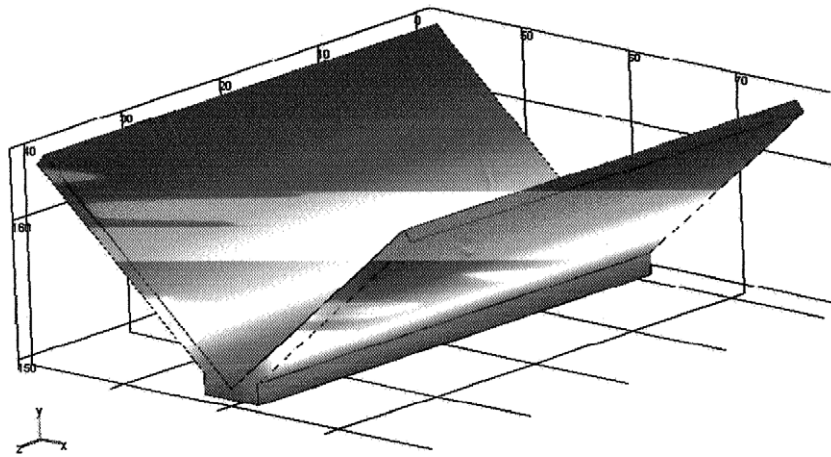


Fig. 12. Temperature field of the fin.

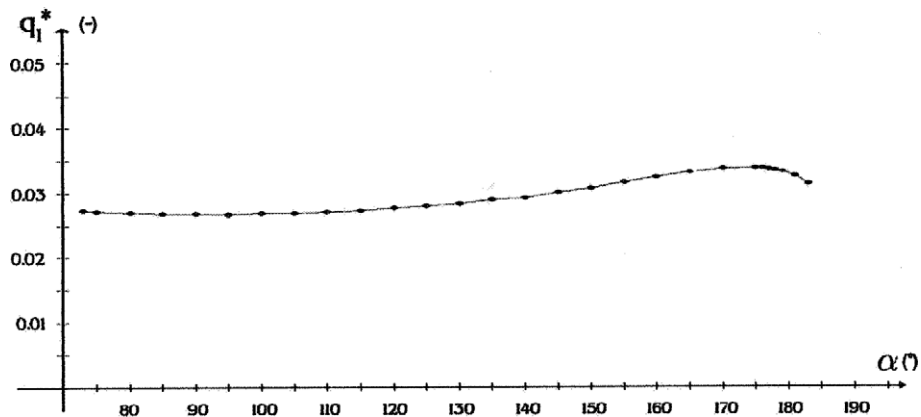


Fig. 13. Dimensionless conductance  $q_1^*$  in function of  $\alpha$ .

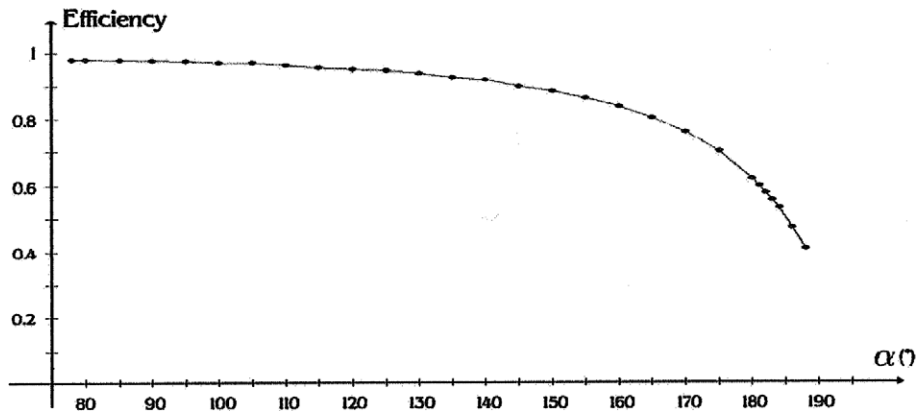


Fig. 14. Efficiency of the profiles in function of  $\alpha$ .

3.2. Partial results

The CFD analysis gave temperature trends in qualitative accordance with the validation section of this research. Fig. 12 reports the pattern of a particular case, qualitatively representative of each case studied.

The complete analysis of the dimensionless conductance, in function of the angle  $\alpha$  characteristic of each configuration, resulted in the trend of Fig. 13. The value of  $q_1^*$  varies between 0.025 and 0.035. Examining the results, one can see that the highest values correspond to angles  $\alpha$  in the vicinity of  $180^\circ$ : this means to configurations very similar to a T-shape. Anyway, reducing the angle  $\alpha$ , holding constant the constraint  $\varphi$ , is associated to a decrease of the horizontal arms length, as shown in Fig. 11. Such behaviour has suggested the introduction of a new performance coefficient able to consider the “weighted” thermal efficiency of the fin. This parameter is defined as the ratio between the actual heat flux exchanged and that exchangeable if the whole surface of the fin were at the same temperature of the root.

In fact, it can be verified that for higher values of  $\alpha$  the temperature gap between the lateral sides of the fin and the root is bigger than for smaller values. The thermal efficiency is so found to be inversely proportional to  $\alpha$ , as shown in the graph of Fig. 14.

The very elevate values of efficiency reached, close to the theoretical limit, have big relevance from a technical point of view. This is particularly true when it is requested the realisation of auxiliary heat exchange devices taking advantage as much as possible of the constrained volumes available.

4. Multiple-fin systems

4.1. Model definition

The efficiency values typical of the configurations with smaller amplitudes of  $\alpha$  have suggested to test the performance of arrays with a variable number of Y-shaped fins. It has been chosen a support of assigned dimension, given by the width  $X'$  of the Y-shaped fin with the best performance in relation to  $q_1^*$  (Fig. 13). As explained in the previous section, this corresponds to a T-shaped geometry with a value of  $\alpha$  approximately equal to  $180^\circ$ .

The number of fins that can be inserted on the support is a function of the width  $X(\alpha)$  typical of each single profile. Its numerical value is expressed in dimensionless terms as follows:

$$X(\alpha)^* = \frac{X(\alpha)}{t_1} \tag{6}$$

and has been plotted in Fig. 15.

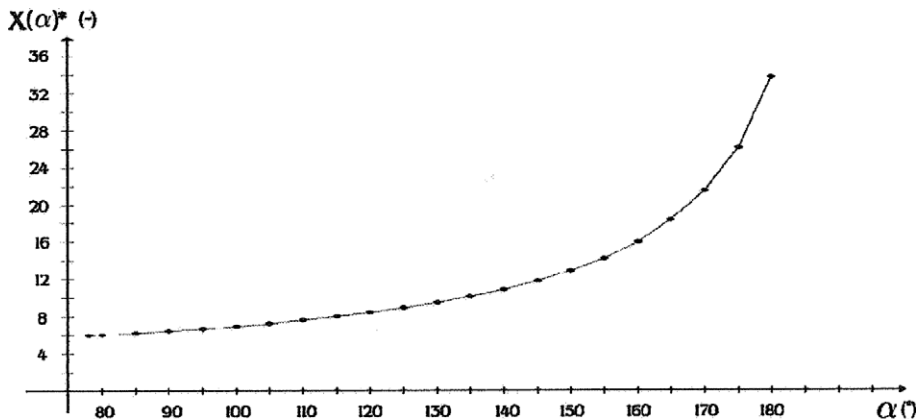


Fig. 15. Length of the fin in function of  $\alpha$ .

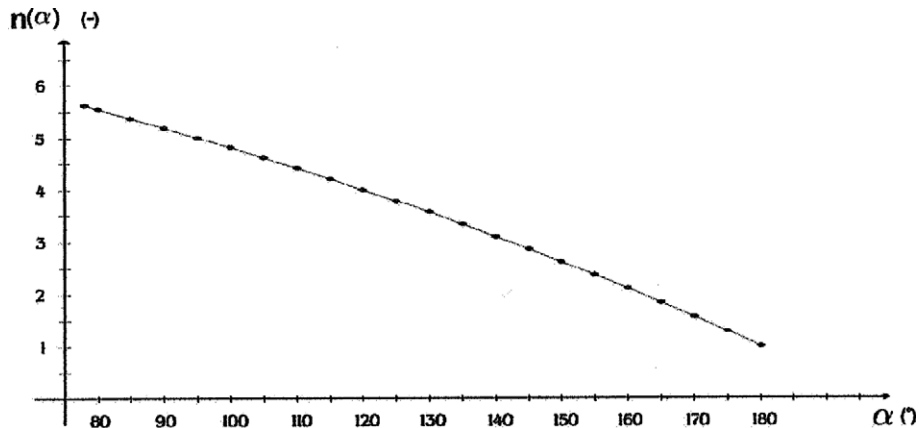


Fig. 16. Number of profiles  $n(\alpha)$  used in function of  $\alpha$ .

The thickness of the root  $t_1$  (Fig. 1) was chosen to make  $X(\alpha)$  dimensionless because it was held constant in all the tests.

Once defined  $X(\alpha)^*$ , the number of fins  $i$  that can be inserted on the support is upperly limited by the integer part of the value  $n(\alpha)$ , obtained as follows:

$$n(\alpha) = \frac{X'^*}{X(\alpha)^*}. \quad (7)$$

The constant  $X'$  was made dimensionless again using  $t_1$ : its exact value is 33.57142. Fig. 16 represents the trend of the function  $n(\alpha)$ .

As it can be observed, the number of profiles inserted on a support cannot be more than 5, as only integer values make sense.

It is now necessary to describe how the fins were joint to the support. Looking towards a practical use of the systems in study, it has been hypothesised to give the system a periodical geometry so allowing for any symmetrical repetition of the modules. To this attempt, it assumes an important role the value of the parameter  $d$ , horizontal distance between the roots of the fins (Fig. 17) defined in dimensionless form as follows:

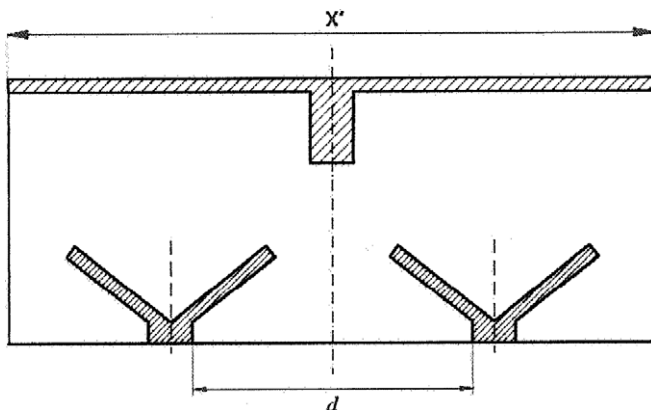


Fig. 17. Modular system in exam.

$$d^* = \frac{X'^*}{i} - 1, \quad (8)$$

where  $i$  is the number of fins used on the support and  $X'^*$  is the dimensionless width of the support, constant in the whole battery of test and equal to 33.57142.

The boundary conditions used in this part of the study were the same as in Section 2.1 to allow a comparison: the temperature  $T_1$  at the root is equal to 373.15 K and the temperature of the fluid  $T_\infty$  is equal to 293.15 K; the thermal conductivity  $\lambda$  is equal to 200 W m<sup>-1</sup> K<sup>-1</sup>. Further detail is necessary about the choice of holding the heat transfer coefficient  $h$  constant, regardless of the spacing between one and another fin, also in the multiple-fin configuration. Such assumption well represents the particular case of a forced convection process in which the refrigerant fluid sweeps the fins along the  $z$ -direction (see Fig. 6) with a very high velocity.

#### 4.2. Results and comments

The computations performed gave in output temperature distributions very similar to those of the previous cases (Fig. 18). It had been again hypothesised the convection coefficient  $h$ , defined in Eq. (5), being constant on all surfaces and equal to that of the single Y-shaped fin in contact with the support. The different combinations of fins analysed were characterised by the trends of  $q_1^*$  presented in Fig. 19 in dependence of the angle  $\alpha$  and of the number  $n$  of fins on the support.

It can be observed the existence of a proportionality among the mean values of dimensionless conductance for each value of  $n$ . The results show noticeable performance enhancement even for small values of  $n$ . This is due to the superimposition of a constant value of  $h$  on every surface, which makes negligible the mutual thermal interactions among the fins. Such an effect is a direct consequence of the efficiency augmentation, inversely proportional to  $\alpha$ , and directly proportional to  $n$ .

The best performance has been obtained therefore in the module with  $n = 5$  (5 fins on the support), characterised by



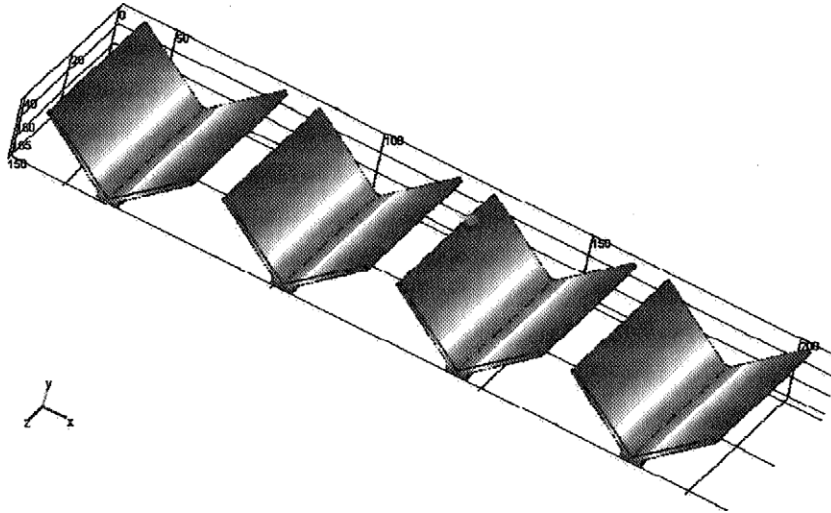


Fig. 18. Temperature trend: 4-fin module case.

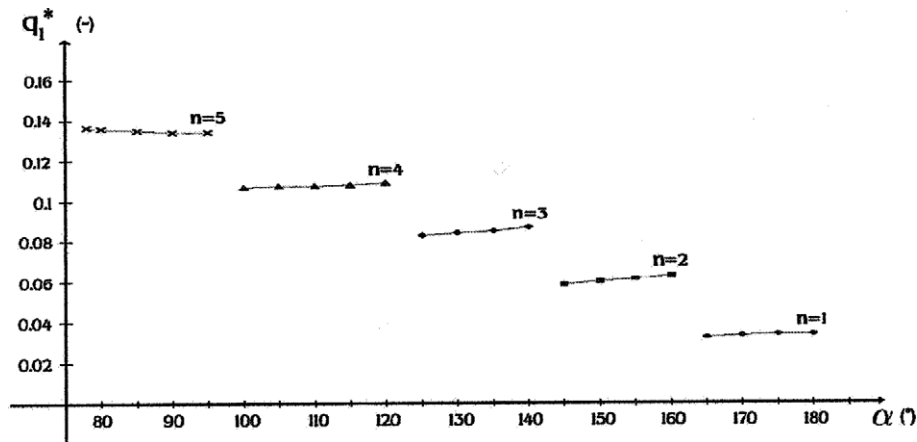


Fig. 19. Conductance  $q_1^*$  in the composite systems.

an angle  $\alpha$  equal to  $78^\circ$ . From a comparison between this result and that of the case with  $\alpha$  equal to  $180^\circ$ , it can be observed an improvement of the heat removal performances. The module offers, being identical the width of the 2 systems, a performance 4.034 times better with respect to that of the original fin.

**5. Conclusions**

This research has faced the problem of optimising the heat removal performance of finned surfaces of particular shape. The aim was that of maximising the thermal flux dissipation in presence of certain constraints. The method applied was based on the use of a numerical CFD code. Such an approach offered a wide range applicability and a managing simplicity which seem very promising for a big variety of problems.

The CFD method, however, had to be preliminary validated before applying to the problem faced. A direct com-

parison with the result obtained by Bejan’s Constructal Theory gave positive answers for a T-shaped fin and allowed for further developments of the study. A first investigation concerned the evolution of the T-shape of the fin towards a Y-shape, obtained varying the amplitude of the angle  $\alpha$  between the two horizontal arms of the fin. The results did not show a particular increase of performance in relation to the dimensionless conductance. Thermal efficiency, instead, showed a significant augmentation in correspondence to a decrease of  $\alpha$ . Moreover, this trend is associated to a reduction of the horizontal width of the fin. Such a behaviour has suggested a new definition of optimisation based both on heat removal and on overall dimension of the Y-shaped fins to obtain systems of multiple fins dimensioned under constrained space. The latter was assigned to be that of the optimised T-shaped case. This approach led to the use of modules of Y-shaped fins, subjected to the same space constraints as in the original case.

The analysis proved that for a number of Y equal to 5, characterised by an amplitude of the angle  $\alpha$  equal to  $78^\circ$ ,

the increase of performances was 4.034 times, in relation to  $q_1^*$  (conductance), with respect to the original case. The research has therefore reached the target obtaining more effective apparatuses in presence of the same spaces allocated.

Such result can have useful industrial applications, as industry seeks ever smaller and ever more effective heat exchangers for its applications.

### Acknowledgement

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